

Solution of Assignment # 3

Moment & Equilibrium

(1)

(1) so the co-ordinates of point A = (0, 5.6, 0)

$$\sim \sim \sim \sim B = (-4.2, 0, 0)$$

$$\sim \sim \sim \sim C = (2.4, 0, 4.2)$$

$$\sim \sim \sim \sim D = (0, 0, -3.3)$$

so the force $\vec{P} = P \hat{j}$

so the force @ cable AB is $\vec{T}_{AB} = T_{AB} \hat{e}_{AB}$

$$\therefore \vec{T}_{AB} = T_{AB} \cdot \frac{\vec{AB}}{|AB|} = T_{AB} \cdot \frac{-4.2\hat{i} - 5.6\hat{j}}{\sqrt{(4.2)^2 + (5.6)^2}} = \frac{T_{AB}}{7} (-4.2\hat{i} - 5.6\hat{j})$$

so the force @ Cable AC is $\vec{T}_{AC} = T_{AC} \hat{e}_{AC}$

$$\therefore \vec{T}_{AC} = T_{AC} \cdot \frac{\vec{AC}}{|AC|} = T_{AC} \cdot \frac{2.4\hat{i} - 5.6\hat{j} + 4.2\hat{k}}{\sqrt{(2.4)^2 + (5.6)^2 + (4.2)^2}} = \frac{T_{AC}}{7.4} (2.4\hat{i} - 5.6\hat{j} + 4.2\hat{k})$$

so the force @ Cable AD is $\vec{T}_{AD} = T_{AD} \hat{e}_{AD}$

$$\therefore \vec{T}_{AD} = T_{AD} \cdot \frac{\vec{AD}}{|AD|} = T_{AD} \cdot \frac{-5.6\hat{j} - 3.3\hat{k}}{\sqrt{(5.6)^2 + (3.3)^2}} = \frac{T_{AD}}{6.5} (-5.6\hat{j} - 3.3\hat{k})$$

And so the point (A) is in equilibrium state then,

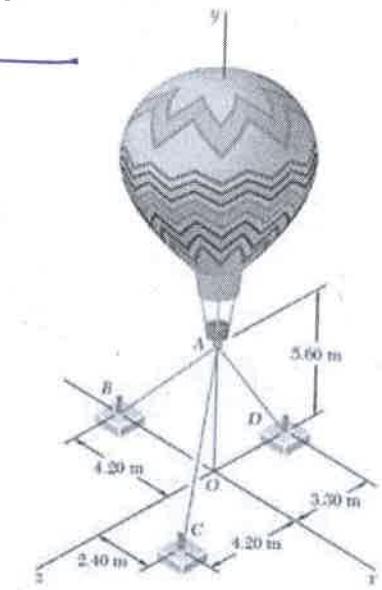
$$\sum f_x = \text{zero} ; -\frac{4.2}{7} T_{AB} + \frac{2.4}{7.4} T_{AC} = 0 \Rightarrow \therefore \underline{T_{AB} = 259 \text{ N "Given"}}$$

$$\therefore \underline{T_{AC} = 479.15 \text{ N}}$$

$$\sum f_y = \text{zero} ; P - \frac{5.6}{7} T_{AB} - \frac{5.6}{7.4} T_{AC} - \frac{5.6}{6.5} T_{AD} = 0 \rightarrow (I)$$

$$\sum f_z = \text{zero} ; \frac{4.2}{7.4} T_{AC} - \frac{3.3}{6.5} T_{AD} = 0$$

$$\therefore \text{from Eqn. (I)} \quad \therefore P = 1031.29 \text{ N}$$



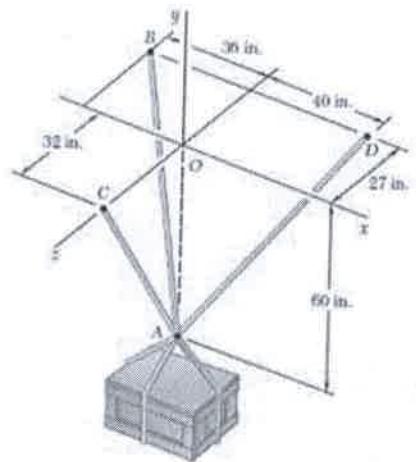
(2)

Given: Each cable can sustain a max. tension = 616-lb.

Req.: the maximum weight of the crate.

Solution:

- The coordinates of Point A = (0, -60, 0)
- " " " B = (-36, 0, -27)
- " " " C = (0, 0, 32)
- " " " D = (40, 0, -27)



In this problem we have 4 unknowns (the weight (w) and tension force in 3 cables)

First: Get the direction of each force.

$$\therefore \vec{w} = -w\hat{j}$$

$$\therefore \text{The force @ cable } \overline{AB} \text{ is } \overrightarrow{T_{AB}} = T_{AB} \cdot \hat{e}_{AB}$$

$$\therefore \overrightarrow{T_{AB}} = T_{AB} \cdot \frac{-36\hat{i} + 60\hat{j} - 27\hat{k}}{\sqrt{(36)^2 + (60)^2 + (27)^2}} = \frac{T_{AB}}{75} (-36\hat{i} + 60\hat{j} - 27\hat{k})$$

$$\therefore \text{The force @ cable } \overline{AC} \text{ is } \overrightarrow{T_{AC}} = T_{AC} \cdot \hat{e}_{AC}$$

$$\therefore \overrightarrow{T_{AC}} = T_{AC} \cdot \frac{0\hat{i} + 60\hat{j} + 32\hat{k}}{\sqrt{(60)^2 + (32)^2}} = \frac{T_{AC}}{68} (60\hat{j} + 32\hat{k})$$

$$\therefore \text{The force @ cable } \overline{AD} \text{ is } \overrightarrow{T_{AD}} = T_{AD} \cdot \hat{e}_{AD}$$

$$\therefore \overrightarrow{T_{AD}} = T_{AD} \cdot \frac{40\hat{i} + 60\hat{j} - 27\hat{k}}{\sqrt{(40)^2 + (60)^2 + (27)^2}} = \frac{T_{AD}}{77} (40\hat{i} + 60\hat{j} - 27\hat{k})$$

Second: Put the equilibrium equations.

$$\sum f_x = 0 \therefore -\frac{36}{75} T_{AB} + \frac{40}{77} T_{AD} = 0 \quad \rightarrow \text{I}$$

$$\sum f_y = 0 \therefore -w + \frac{60}{75} T_{AB} + \frac{60}{68} T_{AC} + \frac{60}{77} T_{AD} = 0 \quad \rightarrow \text{II}$$

$$\sum f_z = 0 \therefore -\frac{27}{75} T_{AB} + \frac{32}{68} T_{AC} - \frac{27}{77} T_{AD} = 0 \quad \rightarrow \text{III}$$

(3)

So, we get 3-equations with 4-unknowns.

∴ we must assume one unknown to obtain others

∴ Assume that the tension at cable $\overline{AC} \Rightarrow T_{AC} = 616 - Ib$

From equ. (II) $\frac{27}{75} T_{AB} + \frac{27}{77} T_{AD} = \frac{32}{68} * 616 = 289.882 \rightarrow (III)$

From equ. (I) $-\frac{36}{75} T_{AB} + \frac{40}{77} T_{AD} = 0 \rightarrow (I)$

∴ $\frac{2052}{77} T_{AD} = 289.882 * 36 \Rightarrow T_{AD} = 391.595 - Ib$

∴ From Equ. (I) $T_{AB} = 423.8 - Ib$ $T_{AD} < 616 - Ib$ ok.

$T_{AB} < 616 - Ib$ ok.

∴ From Equ. (II) we can get the max. weight of the GATE

∴ $W = \frac{60}{75} * 423.8 + \frac{60}{68} * 616 + \frac{60}{77} * 391.595$

$\therefore W = 1187.71 - Ib$

(4)

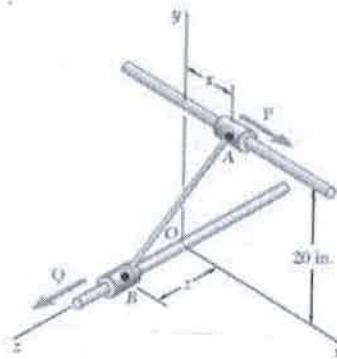
Given: wire length = 25-in

$$\text{Force } Q = 60-\text{lb}$$

$$x = 9\text{-in}$$

Req.: (a) Tension in wire

(b) The force (P)



Solution

\therefore the coordinates of Point A = $(9, 20, 0)$

\therefore the coordinates of Point B = $(0, 0, Z)$

$$\therefore \overrightarrow{BA} = A - B = 9\vec{i} + 20\vec{j} - Z\vec{k}$$

\therefore the magnitude of the vector $|\overrightarrow{BA}|$ = the length of wire

$$\therefore |\overrightarrow{BA}| = \sqrt{(9)^2 + (20)^2 + (Z)^2} = 25 \implies \therefore Z = 12\text{-in}$$

$$\therefore \overrightarrow{BA} = 9\vec{i} + 20\vec{j} - 12\vec{k}$$

\therefore The point (B) is in equilibrium state

$$\therefore \text{the tension in wire } \overrightarrow{T} = T \cdot \hat{e}_{BA} = \frac{T}{25} \cdot (9\vec{i} + 20\vec{j} - 12\vec{k})$$

$$\therefore \overrightarrow{Q} = 60\vec{k} = \left(\frac{T}{25} \cdot 12\right)\vec{k}$$

$$\boxed{\therefore T = 125\text{-lb}}$$

\therefore The point (A) is in equilibrium state

$$\therefore \overrightarrow{P} = P\vec{i} = \left(\frac{T}{25} \cdot 9\right)\vec{i}$$

$$\boxed{\therefore P = 45\text{-lb}}$$

(5)

4] Given: Tension in Cable $\overline{BH} = 450\text{ N}$

Req.: Moment about line \overline{AD}

Solution :

∴ The moment about line equal the moment about point in the line multiplied by the unit vector of the line.

$$\therefore M_{\overline{AD}} = \vec{M}_A \cdot \hat{e}_{AD}$$

$$\therefore \text{the moment } \vec{M}_A = \overrightarrow{AB} \times \overrightarrow{F}_{BH}$$

$$\therefore \text{The Coordinates of Point } A = (0, 0, 0.75)$$

$$\sim \quad \sim \quad \sim \quad \sim \quad B = (0.5, 0, 0.75)$$

$$\sim \quad \sim \quad \sim \quad \sim \quad D = (1.0, 0, 0)$$

$$\sim \quad \sim \quad \sim \quad \sim \quad H = (0.875, 0.75, 0)$$

$$\therefore \overrightarrow{AB} = B - A = 0.5 \hat{i}$$

$$\therefore \overrightarrow{F}_{BH} = \vec{f} * \hat{e}_{BH} = 450 * \frac{0.375 \hat{i} + 0.75 \hat{j} - 0.75 \hat{k}}{\sqrt{(0.375)^2 + (0.75)^2 * 2}} = 400 * (0.375, 0.75, -0.75)$$

$$\therefore \overrightarrow{F}_{BH} = 150 \hat{i} + 300 \hat{j} - 300 \hat{k}$$

$$\therefore \vec{M}_A = \begin{vmatrix} i & j & k \\ 0.5 & 0 & 0 \\ 150 & 300 & -300 \end{vmatrix} = \hat{i}(0) + \hat{j}(150) + \hat{k}(150)$$

$$\therefore \hat{e}_{AD} = \frac{\overrightarrow{AD}}{|AD|} = \frac{1.0 \hat{i} - 0.75 \hat{k}}{\sqrt{(1)^2 + (0.75)^2}} = 0.8 \hat{i} - 0.6 \hat{k}$$

$$\therefore M_{\overline{AD}} = \vec{M}_A \cdot \hat{e}_{AD} = (150 \hat{j} + 150 \hat{k}) \cdot (0.8 \hat{i} - 0.6 \hat{k}) = -150 * 0.6$$

$$\therefore M_{\overline{AD}} = -90 \text{ N.m}$$

